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Research

Geometric Tools in Juan De Álava's Stonecutting Workshop

Abstract. Stonemasonry of the Gothic vault in its totality is based upon geometry of the line, whereas classic stereotomy relies on the comprehensive knowledge of the surface and the highly sophisticated sides of the voussoirs necessary for its vaults. It is obvious that this leap in the art of construction was paralleled and accompanied by an extension of the horizons of geometry. In Spain, it was made possible thanks to the centuries-old tradition of stone building begun in the most remote medieval times and to the presence of outstanding architects or stonemasons such as Juan de Álava, whose professional work surpassed the established limits and provided the art of building with new instruments.

1 Juan de Álava

Medieval stonecutting workshops were places of an extraordinary interest; stonecutters not only tackled the cutting of the various architectural pieces intended for the buildings under construction but also, and more importantly, gave rise to modern geometry. The Gothic vault was originally a rather simple structure, solved with a ribbed crossing, that is, two diagonal arches which intersect in the centre of the vault; nevertheless, this vault, the undisputed main feature of classic French Gothic, took a turn towards complexity. The number of arches multiplied, and so did their intersections on the surface of the vault; the vault began to assume complex forms as a consequence of the web of ribs on which the vault was based. Stonemasons, facing the geometric issues resulting from modern ribbed vaults, had to look for geometric solutions which would allow them to solve the increasingly complex shapes of Gothic vaults: a geometry that was capable of inferring the vault volume from its horizontal projection.

In the first half of the sixteenth century, when we first hear of Juan de Álava (1480? - 1537), Spanish Gothic is at its summit. This architect is undoubtedly one of the outstanding personalities in Spanish Gothic, a style that, far from disappearing in the Renaissance, attained its greatest moment in Spain. Alava's architectural production is considerable and, in particular, achieved a remarkable repertoire of vault like forms which always show his very personal design. When designing his vaults, Juan de Álava avoids the classical Castilian quatrefoil, creating instead a web of ribs that extends along the aisles. This type of vault, completely original in the Iberian Peninsula, is designed only in Álava's workshop.

To build these complex ribbed vaults, this renowned architect resorted to extraordinarily interesting geometric solutions with which he confers shape on a set of architectural shells, one of the most surprising aspects of his work. This article intends to show the advances in geometry achieved by Juan de Álava's construction method. At the beginning of the sixteenth century, the dihedral system of projection was about to make its appearance, and the majority of the tools used for its calculation were already known.

2 Gothic architecture: evolution

The classic Gothic vault is a consequence of the groined vault. Let us recall that the Roman groined vault results from the intersection of two semi-cylinders and, as everybody knows, their crossing produces two elliptical arches. When the vault is made with concrete or rubble masonry, costly wooden centerings need to be built so as to pour on them the material with which the vault is constructed; the elliptical lines of the cross are the natural result of the line left by the two centerings when mortar is poured on them. In the Romanesque style, as in the case of the crypt under the Portico of Glory in Santiago de Compostela, they indulged in highlighting these lines with stone ribs so as to reinforce them, but these reinforcements are absolutely unnecessary in vaults made with rubble masonry. When building the vault with bonded stone, the ashlar beds of the two barrels produce a groined joint along this intersection; in order to solve this joint, there are two options: either to bond the stones which form the groin lines in two directions, entailing extremely difficult stonecutting, or hide this joint with a diagonal rib.

The Gothic vault is the outcome of a variety of achievements occurring simultaneously, of which one takes place when Gothic architects realize that resorting to diagonal ribs, combined with the use of transverse arches as resistant elements, not as mere decoration, can make the very expensive centerings required by Roman and Romanesque vaults unnecessary. The vault can be closed by covering the spaces between the ribs, that is to say the panel, with pieces of smaller vaults without any kind of centering, since the cells start taking shape through small arches that are completely self-bearing and placed between two adjoining ribs. Later in time, with the appearance of new ribs, the vault was broken up into smaller compartments and the panel could be made almost flat between the ribs, making its construction easier and the web considerably thinner. The thick shells of Early Gothic became a thing of the past. In short, the presence of ribs provided an easy solution to the difficult diagonal joint, allowed savings in the panel centerings that can be solved with simple incidental supports and, lastly, represented a powerful aesthetic solution.

A new problem arose: the difficulty encountered when building an elliptical arch in stone since all its voussoirs are different. Let us add that when the vault has a rectangular plan, both barrels have a different span and, in order to keep the vault's ridge line horizontal, the smaller barrel has to be stilted with the unfortunate consequence that the joint of the barrels – the ellipse – no longer occurs in a vertical plane. This circumstance made it necessary to build an elliptical arch whose outline occupies three dimensions. This geometrical problem, difficult from the point of view of construction, was successfully solved by replacing the ellipse with two semicircular arches placed in the vertical plane. However, this choice which, in a certain way, entails a distortion of geometry, has its consequences: first, the vault's crown is higher, so in order to maintain its ridge line horizontal, the semi-cylindrical barrels have to be pointed; second, the vault shells which were cylindrical in the traditional groined vault are no longer so, because, between the curvature of the diagonal arches and the four pointed wall ribs, are formed ruled surfaces shaped like a plow blade, which have to be built with the bonded stones that make up the cell [Frankl 1962: 80; Bechmann 1981: 159; Choisy 1899: 271, 274].

It is known that when the span of the aisles began to reach considerable dimensions – near 15 m. as in most French cathedrals from the thirteenth century on – the diagonal arches are pointed, as Violet-le-Duc explained, so as to reduce strong thrust caused by semi-circular diagonal arches. However, formeret and transverse arches generally keep their original heights so as not to increase even further the height of the perimeter of the aisles, resulting in a vault with slightly inclined ridge lines.

Indeed, the classical Gothic vault was the result of a series of intelligent decisions which led to vaults that were easy to build, economical and satisfactory from an aesthetic perspective. Later on, appeared the tierceron, whose contructive rationale, although justified by Viollet-le-Duc as an elbow-shaped reinforcement of the lintel rib, in fact most likely responds to the need to fragment the cells in order to diminish the length of the beds, thus facilitating its construction and make it less expensive.

Needless to say, from an aesthetic point of view the five-boss vault was a great success compared to the groined vault, and with it the surface of the Gothic vault began to be fragmented into individual compartments, or cells, separated by a grid of ribs.

3 From the semicircular arch to the pointed arch

Let us talk about some advantages of building with diagonal arches. When trying to cover a certain span with a semi-circular arch, there is just one solution: the voussoirs forming that arch will be determined by the radius of curvature of the circumference that describes it. If another arch with a different span is required at the same location, the voussoirs for that second arch would have to have another radius of curvature, and, therefore, they would be different from those of the previous arch; this would be the case with all the arches of distinct span. Consequently when building with semicircular arches, the voussoirs' arrangement, storage and assembly becomes more and more complicated, since the pieces are not interchangeable among themselves; each one requires its adequate placing.

Nevertheless, if, to attain the same span, the arch adopted was pointed instead of semi-circular, the solution is indeterminate, since there are infinite kinds of pointed arches that could cover a given span. It is therefore compulsory to define another parameter to determine the solution, and in medieval times that was the height of the keystone, which made it possible to decide the radius and the centre of curvature, presuming that the arch springs vertically. The degree of indetermination inherent in the geometry of the pointed arch is, at the same time, one of its most powerful advantages. If the plan is to build another arch to cover a different span, it is possible to draw another figure with the same radius of curvature, and, in this case, the variation occurs in the height of the keystone. It is, therefore, possible to draw several arches to cover different spans with the same radius of curvature; in other words, the same voussoirs can be used for different arches. Fig. 1 shows a drawing by Viollet-le-Duc in which this principle is summarized: with the same curvature of a semi-circular arch several pointed arches can be drawn. Applying this principle to a star-shape ribbed vault in which the diagonal arch is a semi-circular arch, the other arches, the tierceron and the formeret could be built with identical curvatures.

Therefore, the use of the pointed arch entails a remarkable advantage for the organization of the works and the transmission of orders among the different members of the workshop constructing the building.

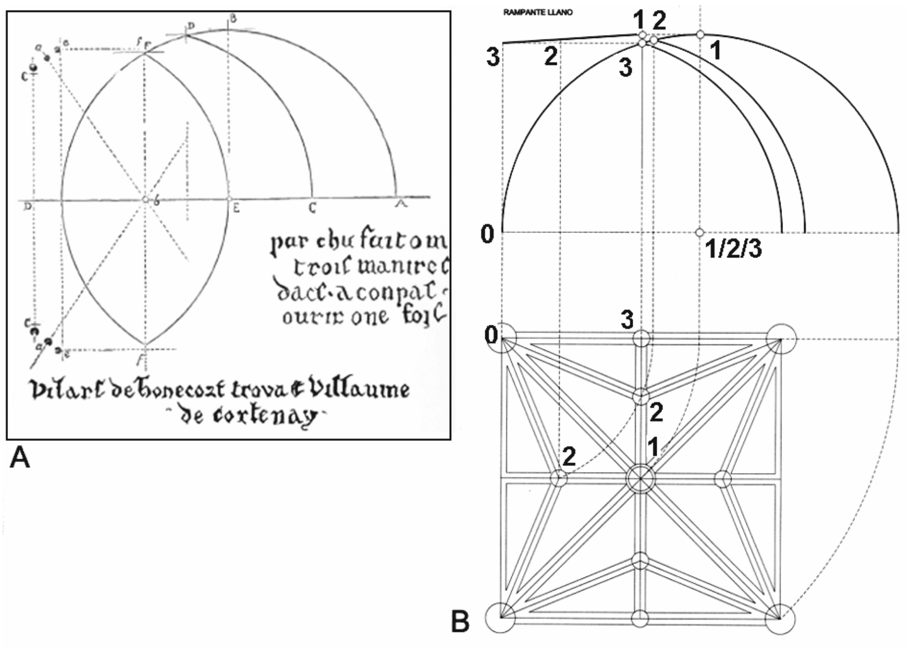

Fig. 1. Drawing by Viollet-le-Duc from a sketch of Villard de Honnecourt, showing how all ribs of a ribbed vault can be drawn from the semicircular arch of the diagonal rib [Viollet-le-Duc 1878: 130])

4 Complexity and standardization

Gothic was an architectural style which succeeded in generating a very articulated aesthetic code, but in addition to its stylistic singularities, it is interesting to notice how the composition rules which made up this style are justified by the existence of a vector which, in a continuous way , encouraged the style to attain higher and higher degrees of complexity. As far as vault building is concerned, everybody knows that Gothic style represents the success of the cross-ribbed vault, undoubtedly one of the most revolutionary inventions in construction history. It is the result of a long process which goes from the simple crossing of ribs in vaults at the beginning of the thirteenth century to the spectacularly complex late Gothic vaults of the sixteenth century.

The principle of complexity as a vector of Gothic evolution was an idea developed in the middle of the twentieth century by Paul Frankl [1962]. This present article aims at elaborating on this issue by adding that the large complexity attained by European Gothic was due to the tools generated in the workshops of medieval stonemasons: geometry and standardization. The aim was to attain the largest complexity with the widest standardization, so as to succeed in building the most complicated ribbed vault with just one radius. The advantages of this technique are obvious: all the voussoirs with which the ribs are built are identical, and so is the wooden centering needed for their construction.

To build a vault's shell by collecting ribs necessitated a strict control of several points in the space which can not be carried out without a largely developed geometry. Each arch has a springing point and must reach a height which is determined by the surface to be built, both points are indispensable to carring out the layout of each arch.

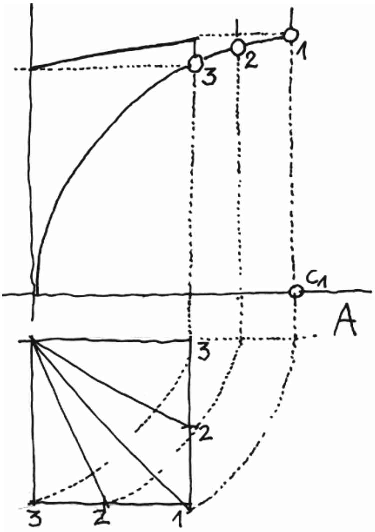

Fig. 2A. All the arches are equal because they coincide with the curvature of the diagonal

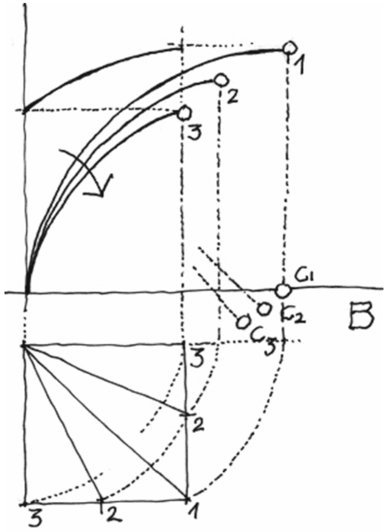

Fig. 2B. The arches are equal because the diagonal arch (semi-circular) bends forward until it reaches the height of the secondary keystones previously determined

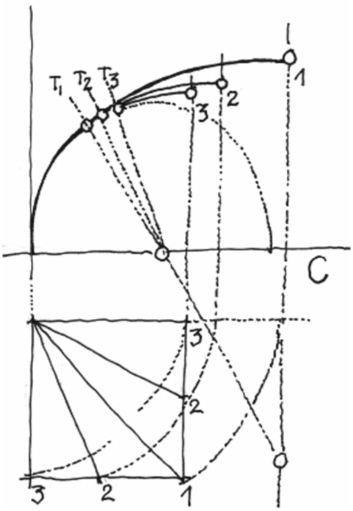

Fig. 2C. When using oval arches, the upper portion of the oval can slide down the lower part. The point of tangency between the two curvatures of the oval moves, but all the arches can be built with the same curvatures

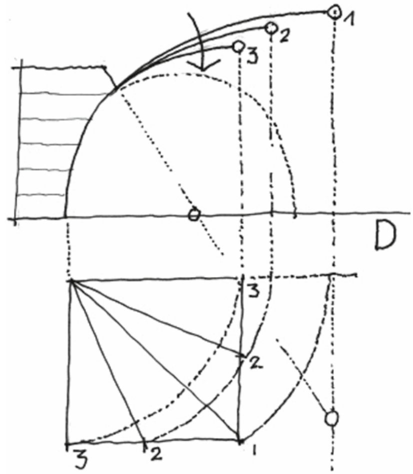

Fig. 2D. The previous drawing can be simplified even more. The lower part of the oval arch does not really exist, it is the solid springing of the vault. The upper part of the oval starts always at the same height, where the springing finishes. From this point, as in case B, the arch formed by the upper part of the diagonal oval tilts to reach the heights of the boss

Fig. 2. Summary sketches containing four different techniques detected in the Spanish Gothic vaults in order to standardize the curvatures of the arches in a vault

This is the moment when the logic of construction imposes a new condition: standardization. This second condition requires the increasing complexity in the ribs to be performed with a minimum of different radii, even with just one radius, when possible. The increasing complexity inherent in the Gothic style, as paradoxical as it may seem, is accompanied by the aim to attain the largest degree of complexity with the greatest simplification; this principle which cannot be applied without achieving standardization of the arches' curvatures.

The evolution of Spanish Gothic architecture in its latest period, in the middle of the transition between the Late Middle Ages and the modern era, is marked by the development of ribbed vaults of increasing complexity in which the ribs and the boss which form them are multiplied incessantly. The most usual model is the stellar groin vault, formed of five bosses (a central boss and four secondary). This composition is the basis of a great number of much more complex compositions in which both the keystones and the tiercerons increase in number to attain compositions that are more and more spectacular. As a general rule, most of these vaults use compass-drawn arches, that is, those having a unique centre located on the impost line of the vault, but vaults constructed with arches having three centers, i.e., ovals, are also very frequent.

The strategies to make all the arches used to build the vault equal are summarized up in the four sketches shown in fig. 2. Let's see now in greater detail the application of each of these cases in the vaults of Juan de Álava.

5 The vaults of "rampante plano" (flat rampant)

Cathedral of Santiago de Compostela, vaults of the cloisters. North nave: built 1521-1527

Let us now analyze the vaults built by Juan de Álava in the north aisle of the cloister of Cathedral of Santiago de Compostela [Castro 2002; Gómez Martínez 1998] (fig. 3)., which has a square plan with all the ribs equal, that is, with identical curvature. After his death, Rodrigo Gil de Hontañón carried on with the construction of the cloister following same principles established by Juan de Álava, as did the successive architects who finished this imposing cloister on the south aisle.

Fig. 3. Cathedral of Santiago de Compostela, cloister, north aisle, Juan de Álava, 1521-27

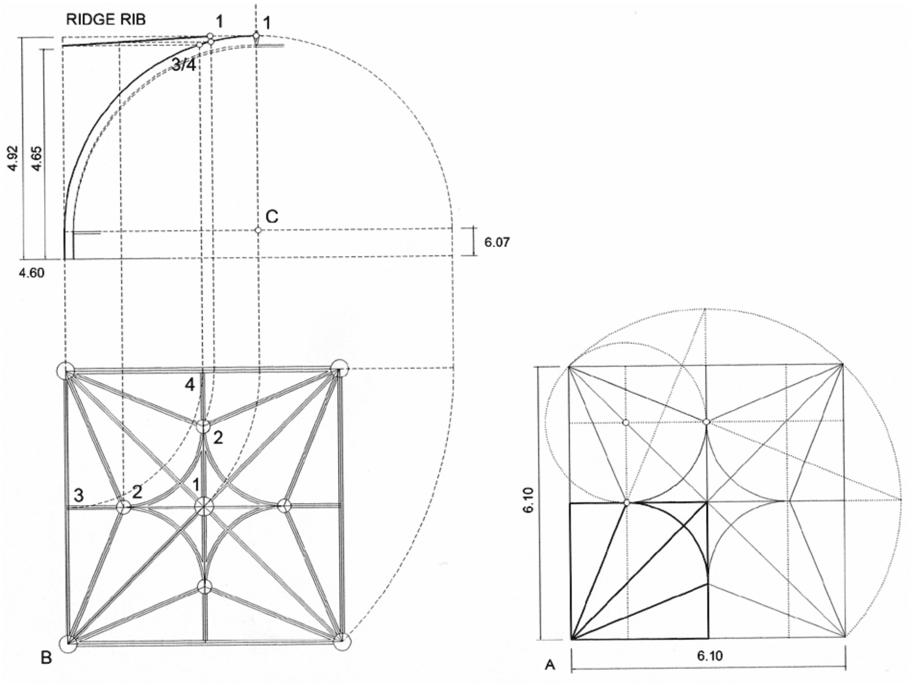

Fig. 4. Cathedral of Santiago de Compostela. Drawing of a cloister vault.
A) plan; B) elevation of the curvatures of its ribs

The plan is a square with a side measuring 6.10 m; over it there is a simple ribbed design which, as we will see, turns out to be spectacular (fig. 4A). The tiercerons are located in a intermediate position between the diagonal and the formeret arches; thus, once the positions of the bosses of the tiercerons are established, vertical and horizontal lines make it possible to determine, on the diagonal lines, the centres of the curves of the subsidiary ribs. Once drawn, they describe a square with concave sides in the centre of the vault. In addition to these ribs, four lierne ribs highlight the ridge line.

The systematic standarization of the sections of the ribs is a remarkable feature in Juan de Álava's vaults [García 1991]. The ribs in this vault all have the same section (around 20 cm), except for the transverse arch, which is slightly thicker (about 25 cm).

Having ribs of equal section help unify the individual bays into a larger whole, the centralized design of each bay becomes less important and the fragmentation into parts is diminished; as a consequence, the whole aisle gives an impression of unity. This visual effect can be stressed by a design of subsidiary ribs linking the vaults. In the cloister of the cathedral of Compostela this is the function carried out by the central lozenge: its curvature, after a straight part of the lierne, goes on in the next part, and so on.

First of all, just by looking at the springing of the vault one can notice a rather marked 60 cm. stilt; second, all ribs are aligned in their intrados; because they are all equal, the aesthetic effect is that of a fluted column. The intelligent combination of both resources creates that impression of balanced classicism derived from this cloister.

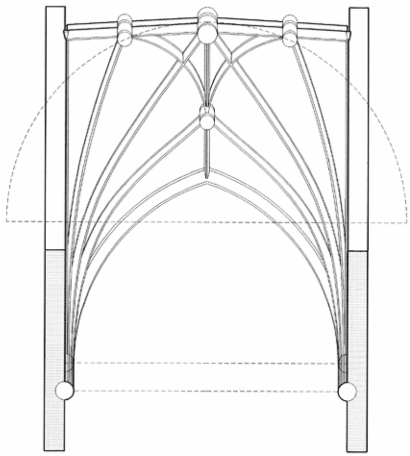

Fig. 5. Cathedral of Santiago de Compostela. Prospective section of one of its cloister vaults in which it is allowed to see its stilted springing, its semicircular diagonal rib and, also, its flat ridge lines

By drawing the elevation of the ribs (fig. 4B), we see that all the arches are drawn with the diagonal's circumference. This strategy, as we know, generates a slightly sloped, in fact almost flat, ridge line (*rampant*). The geometric reconstruction of the lay-out confirms that the diagonals are perfect circular circumferences and the formeret, tierceron and transverse arches are segments of that large diagonal arch; the perspective section shows the gentle slope of the ridge (fig. 5).

Fig. 6 shows the shape of this vault's bosses and, in fig. 7 can be seen the form given to the seven pieces which make up the *tas-de-charges*. With this set of pieces (fig. 8) can be carried out the assembling of one of the vaults (fig. 9), and, by repetition, an alignment of them (fig. 10).

We see how the small concave square works to generate the continuity of the ribs between one vault and the next. The flat rampant in both directions, longitudinal and transversal, has another interesting consequence: the discovery of a fan vault in its simpler configuration: the square trumpet-bell shape.

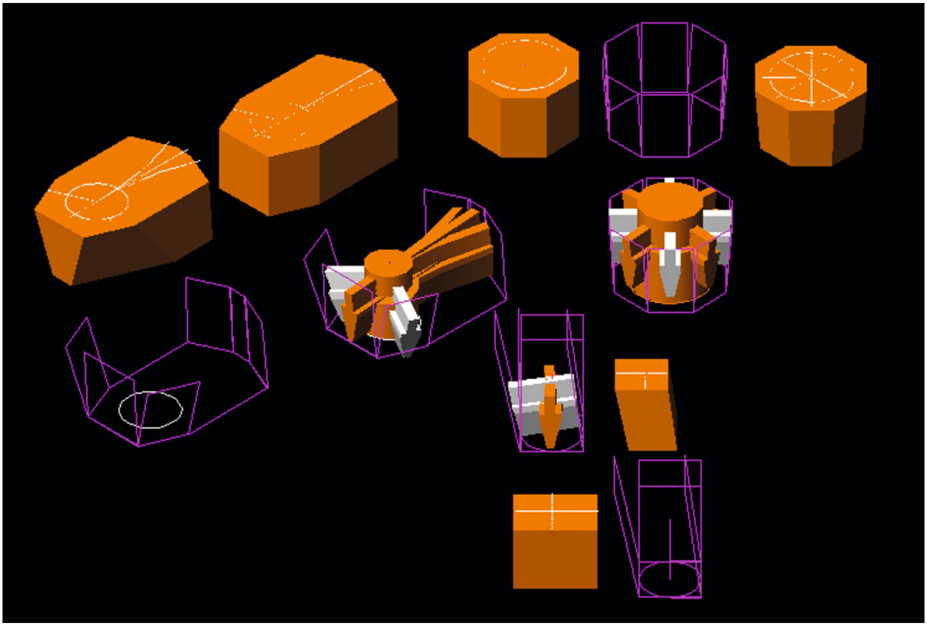

Fig. 6. Cloister vaults in the Cathedral of Santiago de Compostela. In order to cut the boss stones it is necessary to draw them in plan. The horizontal projection of the boss stone is drawn on a flat face of the stone prism; after that, the main shape of the boss stone is obtained by carving away the stone outside the plan drawing

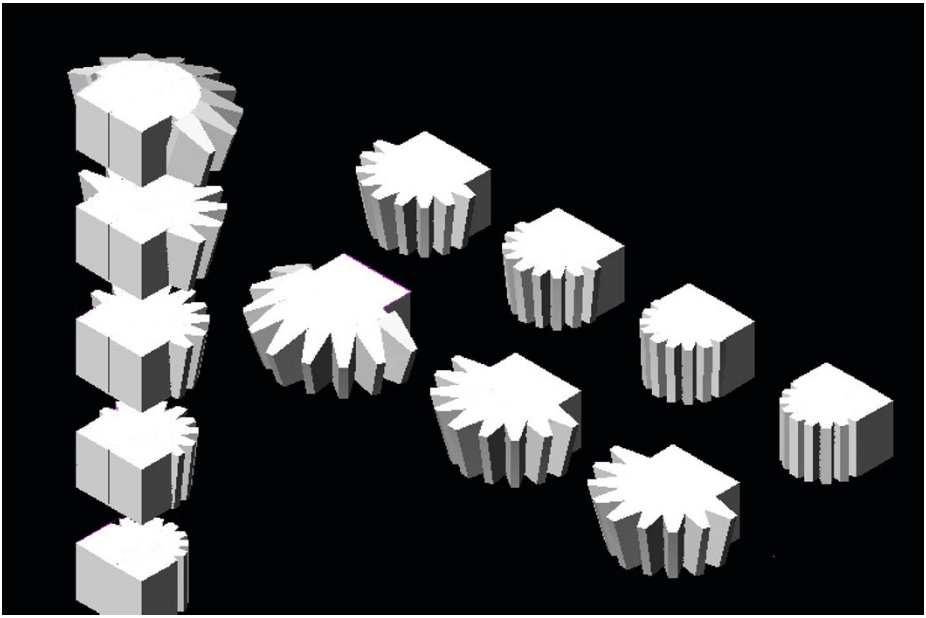

Fig. 7. Cathedral of Santiago de Compostela, cloister.
The *tas-de-charges* is composed of seven blocks of dressed stone

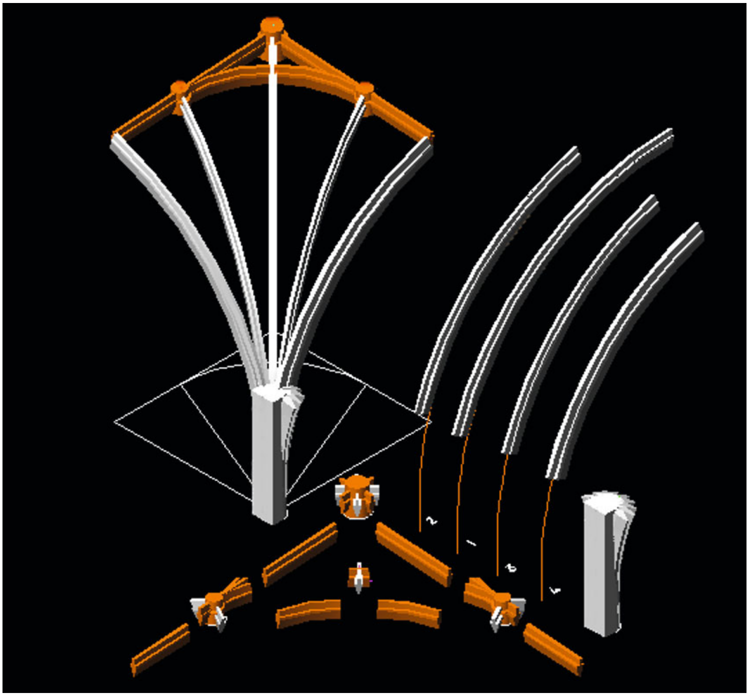

Fig. 8. Cathedral of Santiago de Compostela, a cloister vault. 3D reconstruction of the different pieces that compose a quarter of the vault

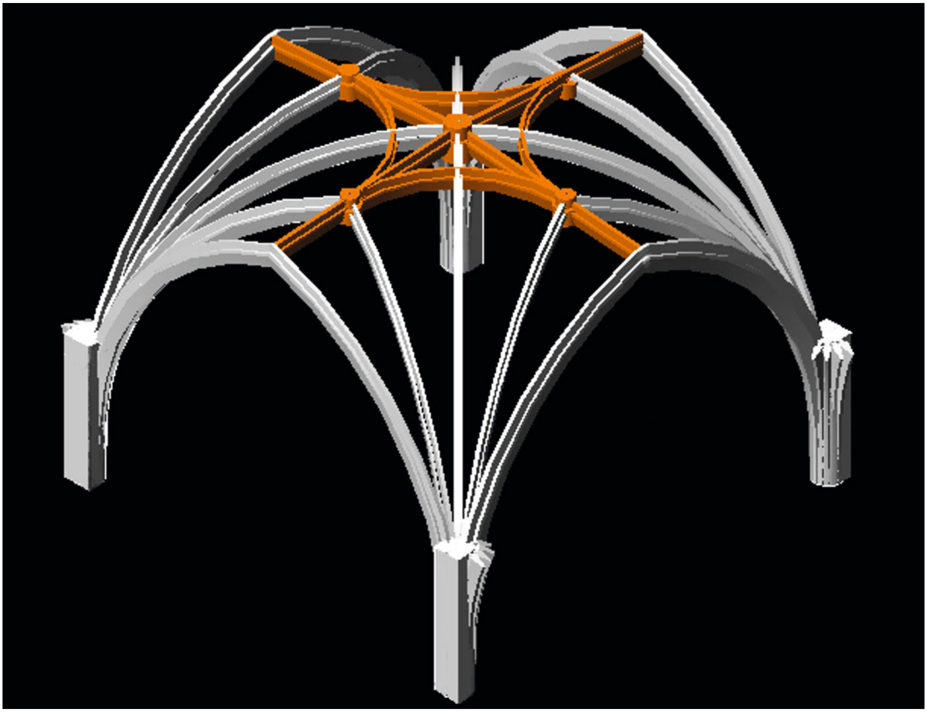

Fig. 9. Cathedral of Santiago de Compostela. Modeling of one of the cloister vaults, in which the specifics features of this vault are clearly shown

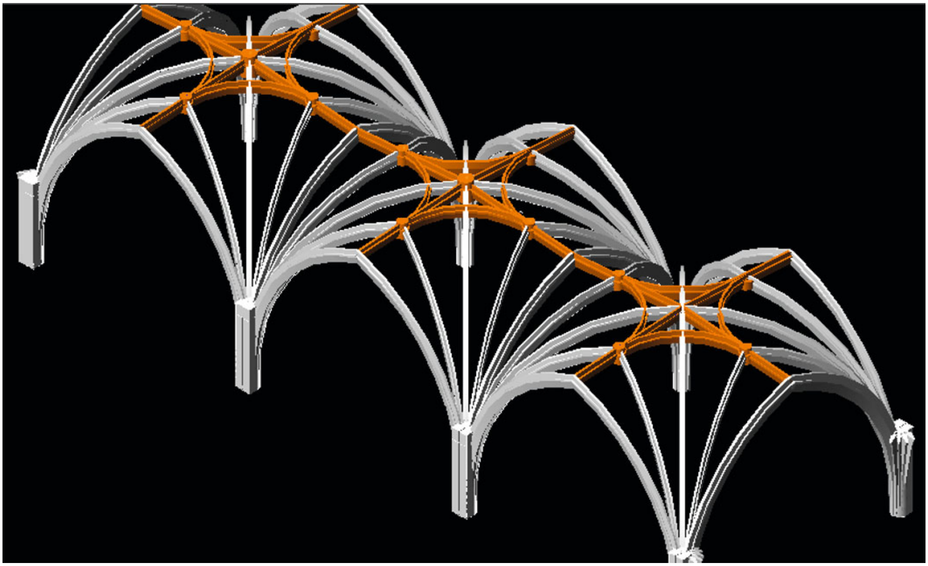

Fig. 10. Cathedral of Santiago de Compostela, cloister. Alignment of the vault. The flat ridge line results in square trumpet-bell shapes, and the subsidiary ribs link the vaults to each other

Fig. 11. Cathedral of Santiago de Compostela, cloister. The corner of the cloister, where the subsidiary ribs enhance the continuity of the vaults

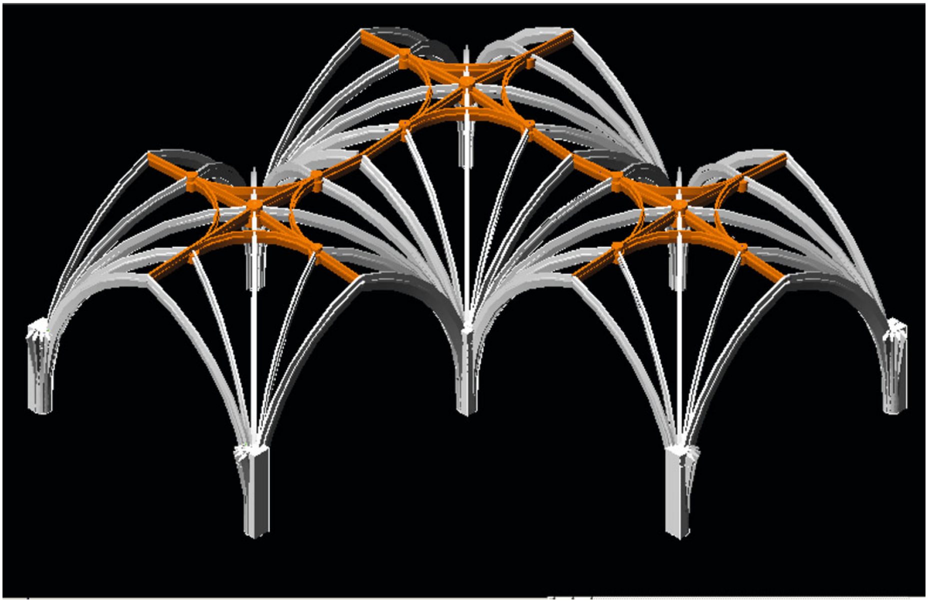

Fig. 12. Cathedral of Santiago de Compostela, cloister. 3D model of a vault turning a corner. All elements with which the vault is composed collaborate to enhance the expression of continutity

Juan de Álava was aware of the formal expressivity of this element, as shown in the corner of the cloister where the vault turns the corner in a most beautiful and elegant way (figs. 11, 12). To our surprise, the architect renounced the opportunity to reinforce this conic effect with horizontal courses that would have created circular stone beds around a vertical axis. Instead, he maintained the Spanish construction tradition of panel bonding and placed the stones *á la française*, that is, as in a groined vault. The cells are made with large stones which span the intervals between the ribs in single pieces, thus requiring therefore no centering or ancillary means of support during construction.

6 Pointed barrel vaults

San Esteban's Monastery, Salamanca. Nave vault. Started in 1526-1533

The vault covering the nave in San Esteban's convent is one of the most spectacular vaults built in Spain. Juan de Álava's masterpiece is an example of his specific way of conceiving the Gothic vault [Castro 2002; Chueca 2001; Navascués 1984] (fig. 13).

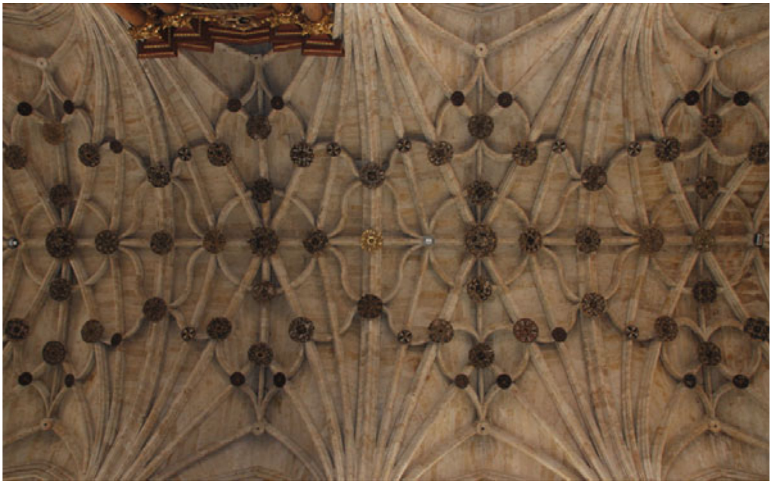

Fig. 13. Convent of San Esteban, Salamanca, vaulting of the nave by Juan de Álava, 1526-33

This vault of this convent church, with a span of 14.50 m, is as wide as the nave of a large cathedral, even exceeding that of Salamanca (13 m). Let us recall that the cathedral of Plasencia, at 17.30 m, has the third largest span in Spanish Gothic, following those of Gerona (23 m) and Palma (20 m). We can therefore attribute to Juan de Álava the merit of having achieved some of the largest spans in Spanish Gothic. After Álava's death, the construction of both Plasencia and San Esteban was carried on by Rodrigo Gil de Hontañón.

The nave of San Esteban is divided into segments of roughly 7 m, so each segment is a rectangle *duplo*, that is, twice as long as it is wide. The complex design of its rib work is perfectly determined in an orthogonal 8 x 4 grid (fig. 14), making it possible to fix the position of the double bosses of the tiercerons along the major axis and of the simple tiercerons along the minor axis. This grid also determines the dimension of the central circle. Drawing from these points, by alignments, are obtained the other bosses and crossings, as is the case of bosses 4 and 2.

When calculating the curvatures of the arches (fig. 14B), the skill with which this vault is designed becomes evident. We soon realize that its diagonal arch is exactly a circular circumference with its horizontal plane at the height of the capitals.

Later, when drawing the *rampants*, we were able to confirm that, towards the major axis, the ridge line is slightly curved and presents a steep slope to the formeret arch of almost 2 m high; in contrast, towards the minor axis, the ridge line is almost flat. As we know, this combination of horizontal *rampants* along the nave axis and a steep slope in the transverse direction, is what creates the tunnel effect in this type of vaults.

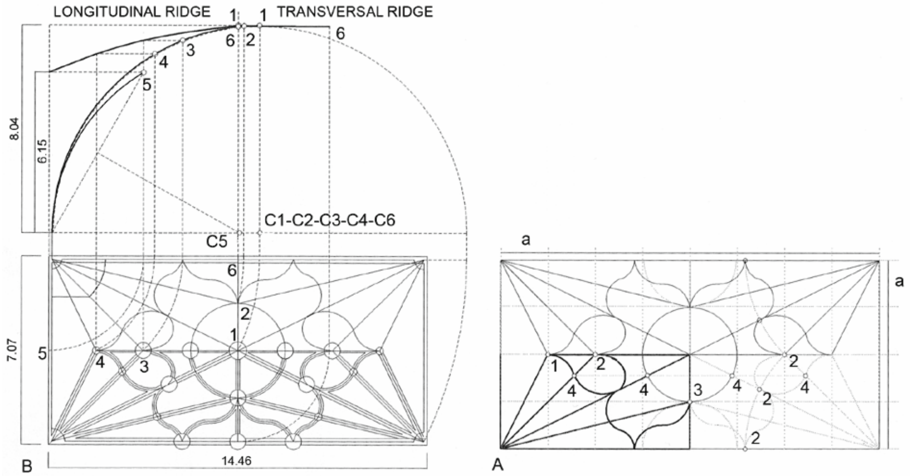

Fig. 14. Convent of San Esteban, Salamanca. Drawing of a cloister vault: A) plan; B) arches curvatures elevation. Note that the curvatures of its arches coincidence perfectly with the semicircular diagonal arch; only the curvature of the formeret arch seems to be different

When calculating the elevation of the other arches, to our surprise, we realized that tiercerons 4 and 3, as well as tierceron 2 and the transverse arch, coincide with the semi-circular circumference of the diagonal (fig. 14B); the only different arch is formeret 5. As frequently in Álava's work, the shape of the transverse pointed arch comes very close to being a semi-circular circumference; its centre is situated next to the arch's vertical axis, at approximately an eighth of half of its span. Therefore, in spite of this vault's apparent complexity, all its main ribs have identical curvatures. Here again we find an insistence on systematizing and standardizing the form of the arches.

The three-dimensional drawings of a segment in particular (fig. 16) and of the nave as a whole (fig. 17) allows us to visualize the huge spatial web made up by the bosses and ribs; these three-dimensional grids are the core of the vaulting works of late Gothic. Upon them are easily laid out the thin panels which, as regards the work of Álava, are always bonded *á la française*.

The *combados*, or subsidiary ribs, play a very important role. It is an extremely original configuration which has an influence on Juan de Álava's desire to create rib designs that intertwine between the different segments. Keeping the star shape in each segment of the nave, the subsidiary ribs generate links which confer unity and flow on the entire nave; the result is the kind of web design found so frequently in the vaults of central Europe. The perspective section in fig. 15 shows the nave's ridge line in transverse, which is a consequence of the architect's desire to standardize the curvatures of the arches. One can see, in fig. 15, that the central boss is located at the top of the semi-circular diagonal arch, indicated by the dotted line.

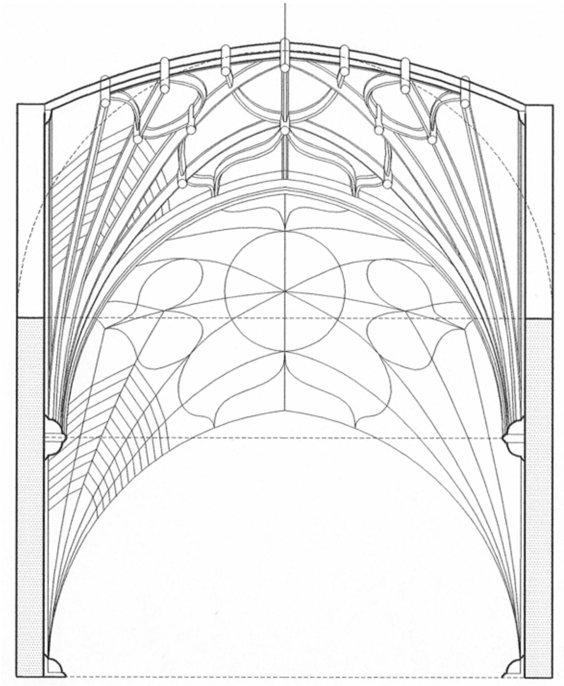

Fig. 15. Convent of San Esteban, Salamanca. A perspective section of a vault through its transverse ridge line makes it possible to see clearly the curvature of the ridge line and how the central boss stone is located at the highest point of the semi-circular diagonal rib

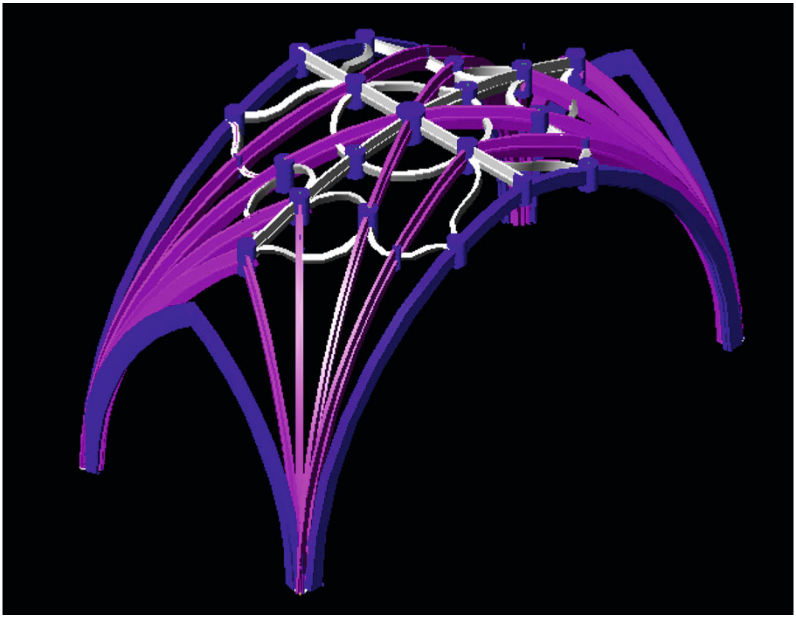

Fig. 16. Convent of San Esteban, Salamanca. 3D model of the spatial grid of ribs that compose the vault of a single bay

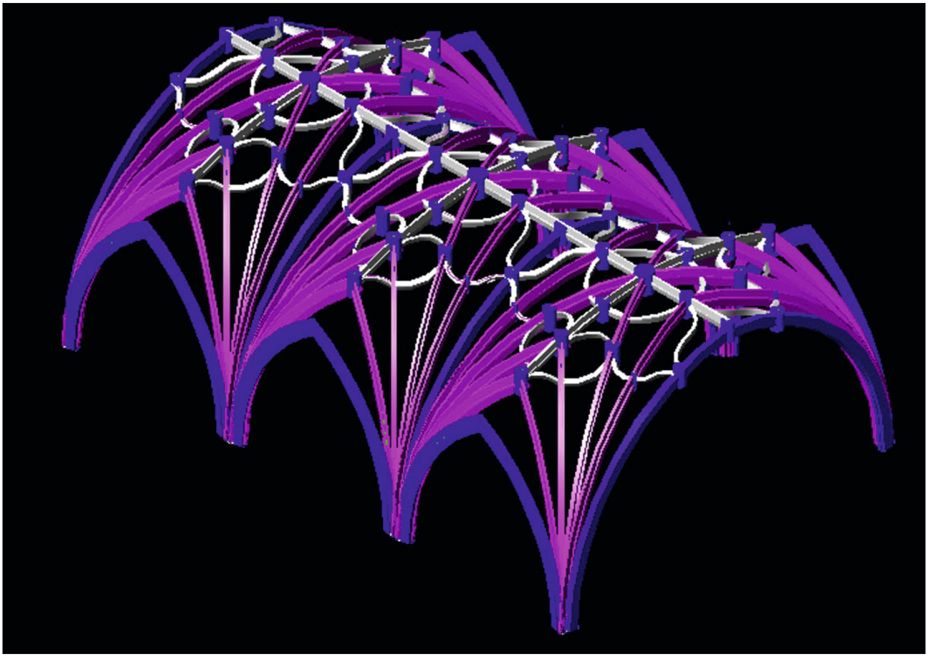

Fig. 17. Convent of San Esteban, Salamanca. 3D reconstruction of the spatial grid of ribs of the nave, probably the most spectacular vault ever built in Spain

Cathedral of Plasencia, Cáceres. Presbytery vault. Started in 1522

The vault covering the presbytery of Plasencia Cathedral [Araujo 1997; Castro 2002; Navascués 1983] is also very spectacular (fig. 18). The front is formed with two vaults, one in the presbytery and one in the apse, intertwined by a daring design of subsidiary ribs based on concentric circles. In linking with one another, the secondary crossings generate a very elegant waving movement.

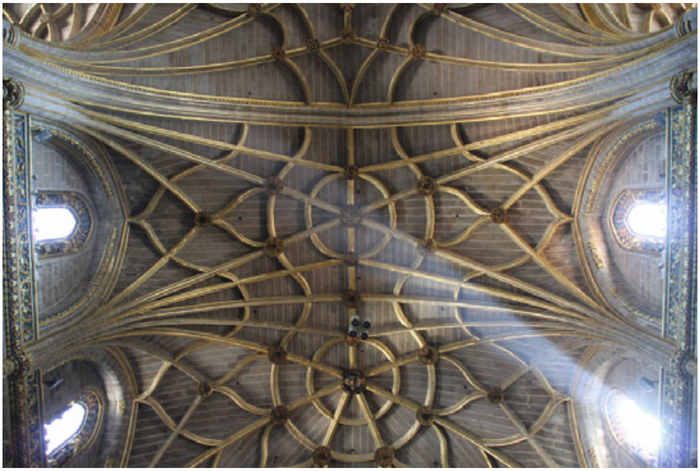

Fig. 18. Cathedral of Plasencia, Cáceres. Vault over the presbitery, Juan de Álava, started 1522

In order to achieve that effect, Álava thins and standardizes the sections of the arches as much as possible, especially the transverse arch which, because it is practically equal to the others, prevents the vault from being fragmented in segments.

Besides, with its 17.30 m span, this is one of the biggest Gothic vaults built in Spain. The width of the bay studied (7.34 m) means that its plan rectangle is remarkably elongated, exceeding the proportion 2 : 1. No doubt a division into such long rectangular bays has important structural consequences since, because the bays are narrower, there is an increase in the number of transverse arches and, on the outside, in the number of buttresses. Álava's great skill is evident when he adopts this solid structural layout to emphasize the lightness and elegance of the vaults [Rabasa 2000].

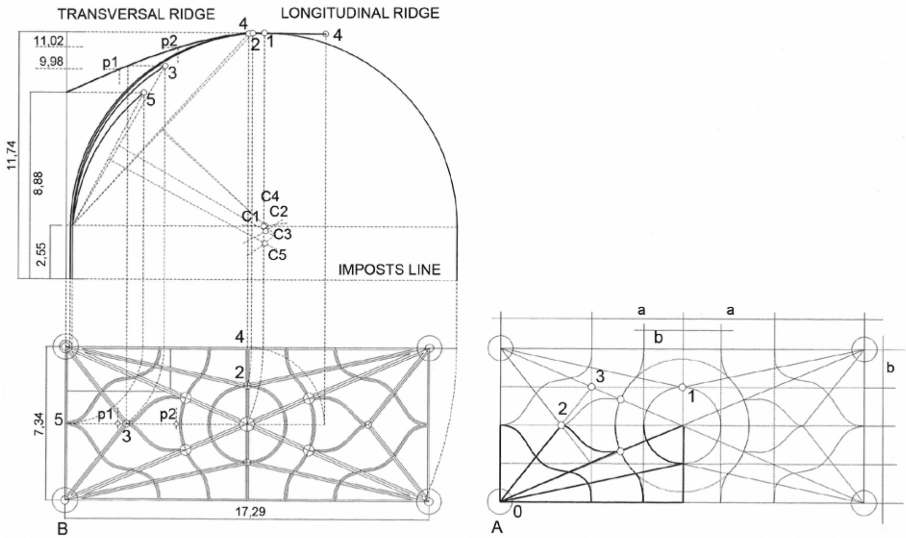

Fig. 19. Cathedral of Plasencia, Cáceres. Drawing of the vault located over the presbytery: A) plan; B) elevation of its arches. Notice the coincident curvature between the tierceron rib (2) and the transvers arch with the semicircular curvature of the diagonal rib. The curvatures of tierceron rib (3) and the diagonal have been made identical curvature by tilting them forward

As far as its morphological design is concerned, one can easily see that a 4x4 grid determines the locations of the most important bosses (fig. 19A). When looking at the four segments in which the small side can be divided, we realize that the first segment fixes the position of tierceron 1, the central circle's diameter and the links with the adjoining vault. The intersection with the vertical grid determines point 3, which in turn determines the position of tierceron 2. The remaining ribs seem to have been designed in a more random way.

In order to calculate the curvatures of the arches, we start by bringing up the diagonal (fig. 19b), which immediately shows that the two diagonal ribs are two extraordinarily stilted semi-circular circumferences, their curvature beginning 2.54 m above the capitals. This stilt contributes to create that impression of slenderness and lightness present in that vault.

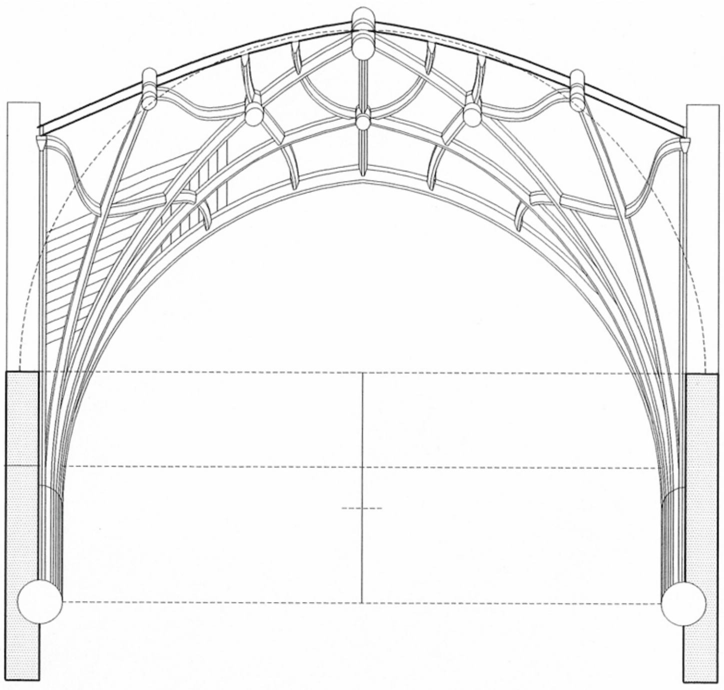

Fig. 20. Cathedral of Plasencia, Cáceres. Prospective cross-section showing the stilted springing as well as how the central boss stone is placed in the top of the semicircular diagonal rib (shown as a dotted line)

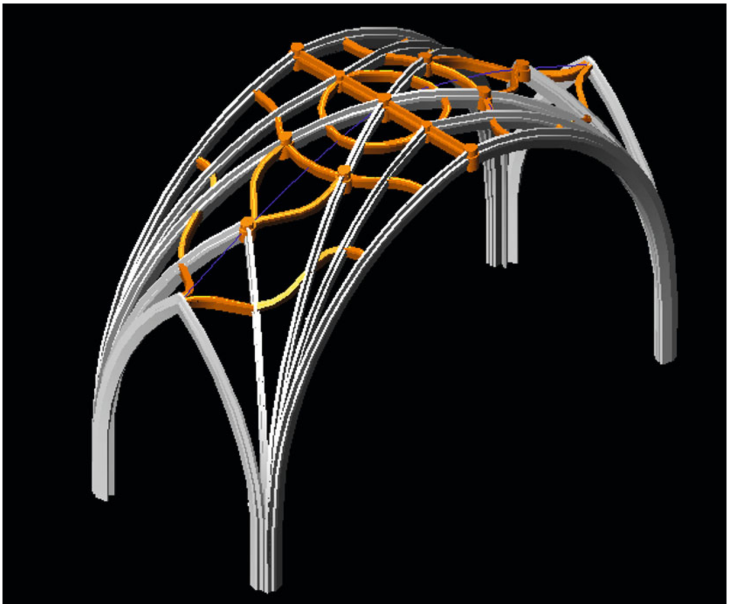

Fig. 21. Cathedral of Plasencia, Cáceres. 3D model of the vault over the choir drawn from the lines obtained geometrically. The image shows its pointed barrel vault shape

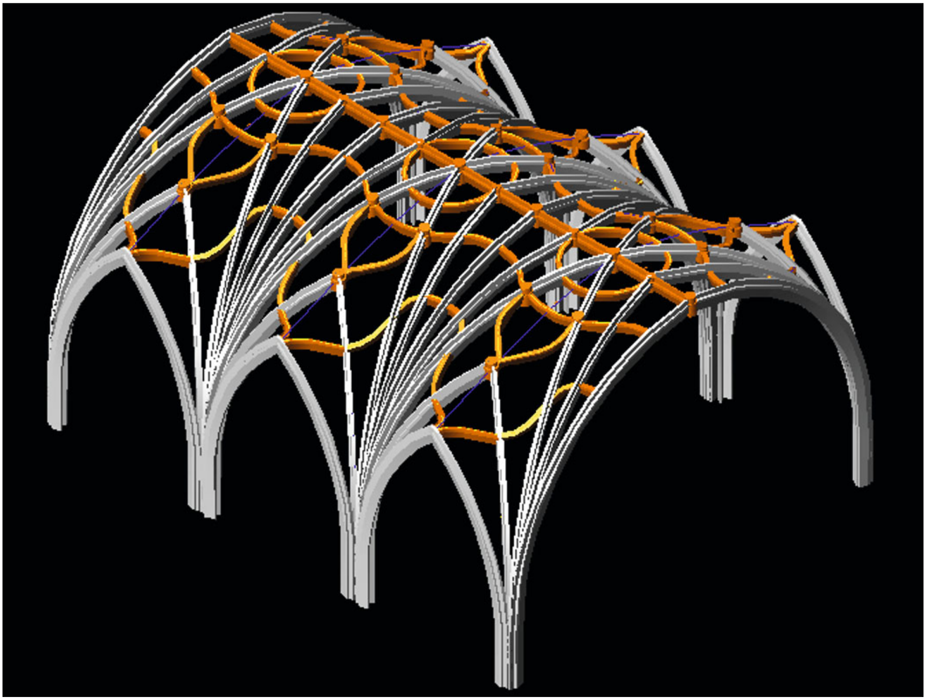

Fig. 22. Cathedral of Plasencia, Cáceres. Hypothetical reconstruction of a nave from the vault of the presbytery. This 3D model shows the intrincate grids of ribs that Juan de Álava was able to conceive

As he did in San Esteban, here too Álava adopts a design in which the transversal ridge line slopes steeply towards the formerets; in contrast, the ridge line along the nave axis is almost horizontal. As in previous cases, this configuration will result in creating a rather marked tunnel effect.

Unlike in San Esteban, in the Cathedral of Plasencia, when drawing the formeret and tierceron 3, according to the line of the transversal *rampant*, we see that neither of the two arches adjusts to the drawing of the diagonal semi-circular circumference; let us recall that each of the three arches is different. Nevertheless, bearing in mind the rational principles evident in Juan de Álava's work, we prefer to advance the hypothesis that the formeret as well as the tierceron 2 have been designed with the same curvature as the diagonal semi-circular circumference. If that hypothesis were correct, the centres of these arches would have to be below the impost level, and, therefore, the arches cannot be tangent to the vertical line. It is impossible to detect this feature visually, and it is further masked by the strong vertical stilt.

In the other direction the problem is easier, since a very slight slope of the rampant locates the bosses of tierceron 2 and the transverse arch in the curve of the semi-circular arch. The transverse arch assumes the form of an almost semi-circular pointed arch, with its centre placed next to its vertical axis, more or less at an eleventh part of the half of the span. Thus, if the hypothesis previously presented was correct, the whole of the vault could be carried out with just one arch.

Finally, a spectacular design of subsidiary ribs based on two concentric circumferences and some helpful ogees establishing four linking points with the apse's vault contribute to the creation of the waving movement of this rib work. The cells, as it is typical in Alava, are bonded *à la francaise*. The section in perspective in fig. 20 showse the rampant line and how the vaults rise on vertical stilts, most probably to allow them to reach the height of the vault of the transept.

The modeling of the vault of the presbytery in the Cathedral of Plasencia (fig. 21) again reveals Juan de Álava's mastery in conceiving and, later, building these three-dimensional webs. Fig. 22 shows how this rib work could have extended along the nave; although never built, it reveals the singular grid frames present in Juan de Álava's imagination.

7 Lierne vaults

It could be said that the main rib of the Gothic ribbed vault is the lierne, since its ridge line determines the height of the bosses which are situated on it. Curiously, in English Gothic this rib is always strictly horizontal, which places all the bosses along its run at the same height; as a consequence, the curvatures of the ribs with which the vault is built are subject to that inevitable starting condition. The visual effect of these sequences of bosses, in both the longitudinal and the transversal direction, is one of the most characteristic features of English Gothic style.

When English vaults accumulate tiercerons and countertiercerons, the conditional of the horizontal lierne results in a large collection of unequal arches. In order to rationalize the number of different arches, English Gothic architects developed the pointed arch with four centers known as the "Tudor arch".

There is, therefore, a significant difference between the "*flat rampant*" vault and the vault in which the lierne line is strictly horizontal. In the former, the diagonal arch has a semi-circular circumference, and the tiercerons are segments of this arch, that is, they are pointed arches with one center, whereas in the latter, the diagonal arch has three centers, which means that it is an oval, and the tiercerons are formed with this same oval, maintaining the springing arch and sliding the upper part of the oval onto the lower part in such a way as to keep the ridge line of the vault strictly horizontal. Thus, it is not an oval vault, since the rampants in it are not round.

Although the term "lierne vault" has a broader meaning in England, we have adopted this denomination for this particular type of vault and distinguish it from the "*flat rampant*" vault.

San Esteban Monastery, Salamanca. Cloisters vaults. Built around 1533

Lierne vaults are apparently simple, but have a layout which conceals surprising features. One example is the vault in the Convent of San Esteban, Salamanca (fig. 23).

The cloister vault in San Esteban is actually a square vault with five bosses. The two tiercerons (see fig. 24A) are located in the position of the diagonal, between the transept and the formeret arch; we know perfectly well how to establish the position of this keystone through the circumscribed circumference and the straight line 1, 0 which, when crossing with the axis, determines point 2. In the center of the vault is a subsidiary rib whose diameter is set by the length 2-3, that is, the distance which separates the boss of the tierceron and that of the formeret. In each of its corners is a subsidiary rib that is one quarter of a circle, whose measure is determined by the alignment A, B, C; this is an open subsidiary rib, that is, one whose purpose is to connect with that of the adjacent bay in order to produce the linking of the ribs that is so characteristic of this architect's work.

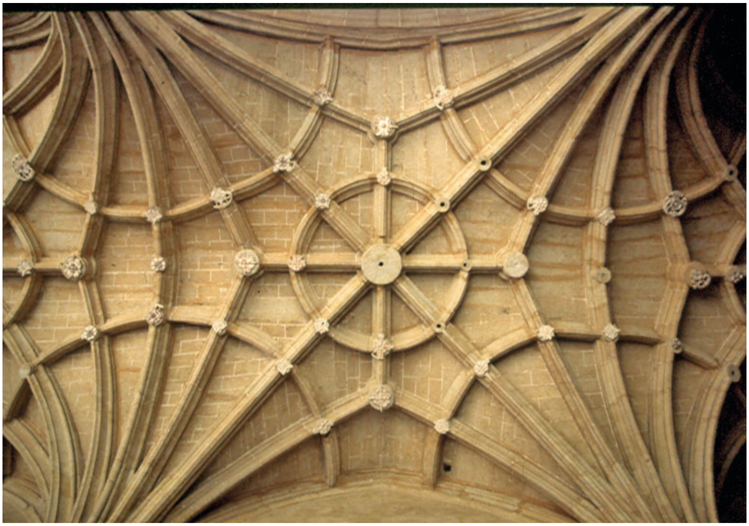

Fig. 23. Convent of San Esteban, Salamanca, vaults in the cloister by Juan de Álava

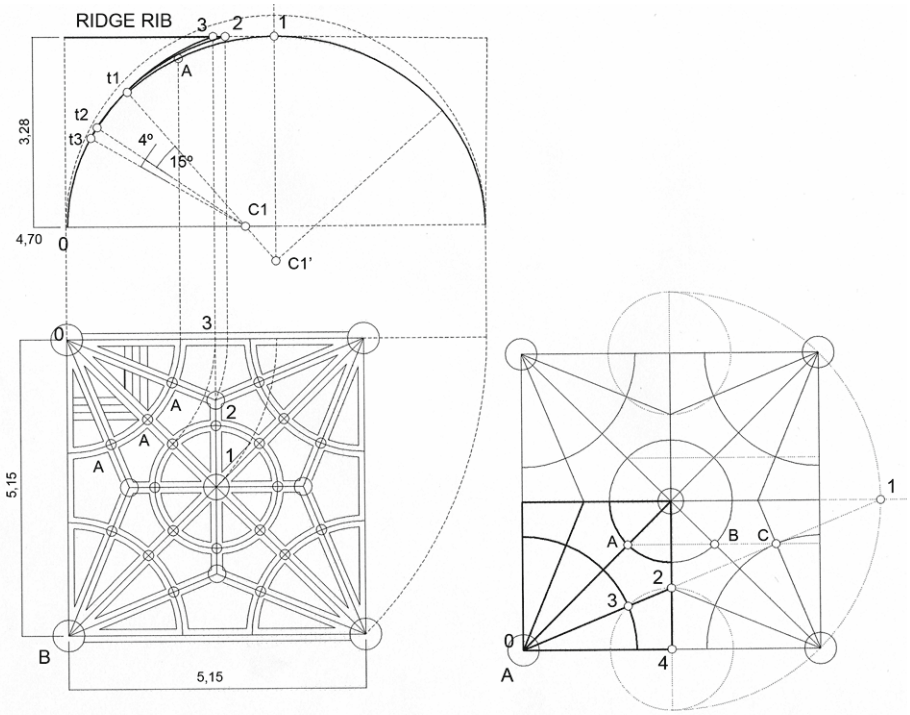

Fig. 24. Convent of San Esteban, Salamanca, cloister. Drawings of one of its vault: A) plan; B) elevation of the curvatures of its ribs. Observe that the diagonal arch has three centers and that the ridge line is a horizontal straight line. The other ribs are obtained by sliding the upper part of the oval diagonal onto the lower part, so that the curvatures of all ribs are identical

When laying out the arches (fig. 24B), we immediately note that the boss of the diagonal arch falls below the theoretical semi-circular arch; this is therefore a surbased arch which has to be laid out with three centers. Using the measurements taken on site, we have been able to draw the oval 1 and centers C1 and C1′, which appear to fit the actual built arch perfectly.

We then proceed to draw the tiercerons, bearing in mind that they must reach the height determined by the vault's horizontal rampant. Obviously, the tierceron and the formeret could be pointed arches different from each other, but it is more advantageous to build them with the same diagonal arch, and thus to succeed in having the three arches making up this vault with the same layouts. To perform that, it suffices to rotate the upper section of arch 1 15° onto its lower section, so that boss 1 moves onto position 2, and the tangency point of oval 1 passes to 2. The boss of formeret 3 is carried out the same way, rotating the upper part of oval 4 onto the lower part. Proceeding this way, one obtains three different arches, which, however, have in common the upper and lower part; needless to say, this process resulted in significant simplification in cutting the voussoirs and the making the centerings.

The vault, being surbased, has a flattened ridge, which makes it possible to place the central wheel of subsidiary ribs at its summit, its circumference broken into few segments. This is also the case of the corner rib which, thanks to the fact that the three vaults arches are made from the same oval, can describe a continuous and horizontal circumference. The panels of the cells of the vault are bonded *á la française*. Fig. 26 shows some of the most outstanding circumstances of the vault: its horizontal rampant, the location of the boss below the semi-circular arch and its groined panel-work. In the three-dimensional image of a section (fig. 26), one can clearly see the volume obtained.

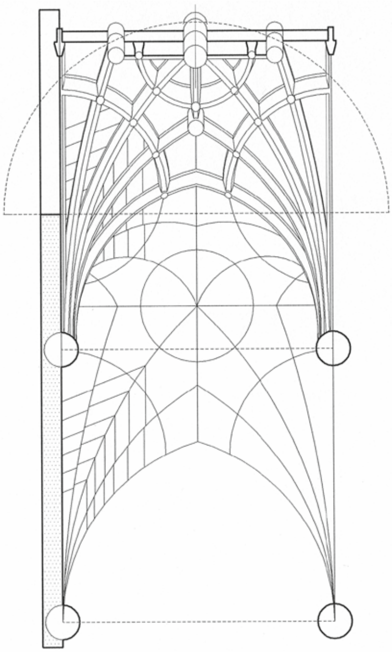

Fig. 25. Convent of San Esteban, Salamanca, cloister. Prospective cross section of the vault showing its horizontal ridge line and the position of its central boss stone. The stone masonry of its web is placed *à la francaise*

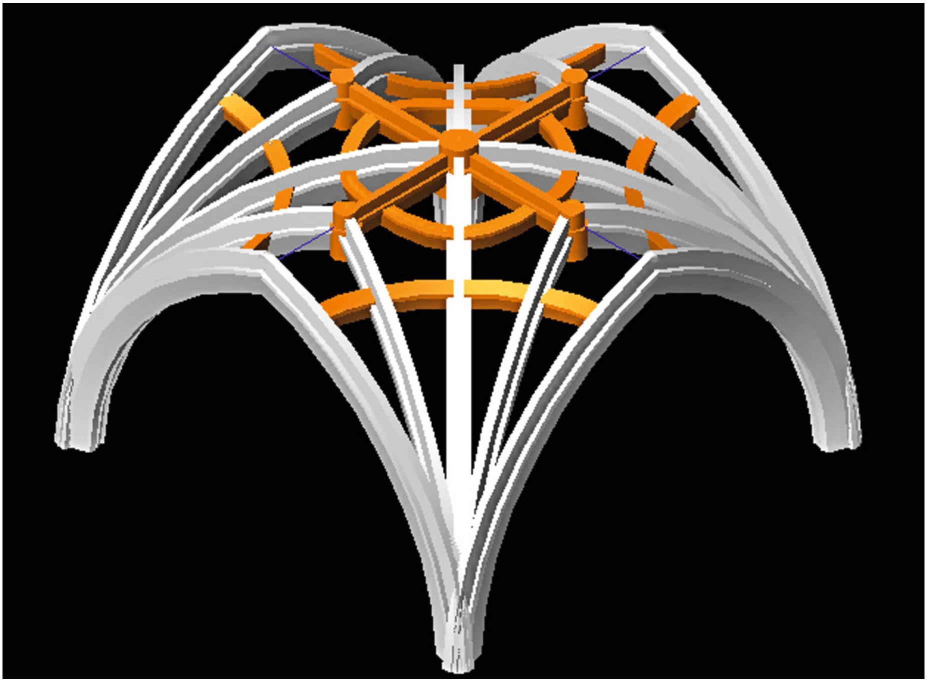

Fig. 26. Convent of San Esteban, Salamaca, cloister. 3D drawing of the resulting vault

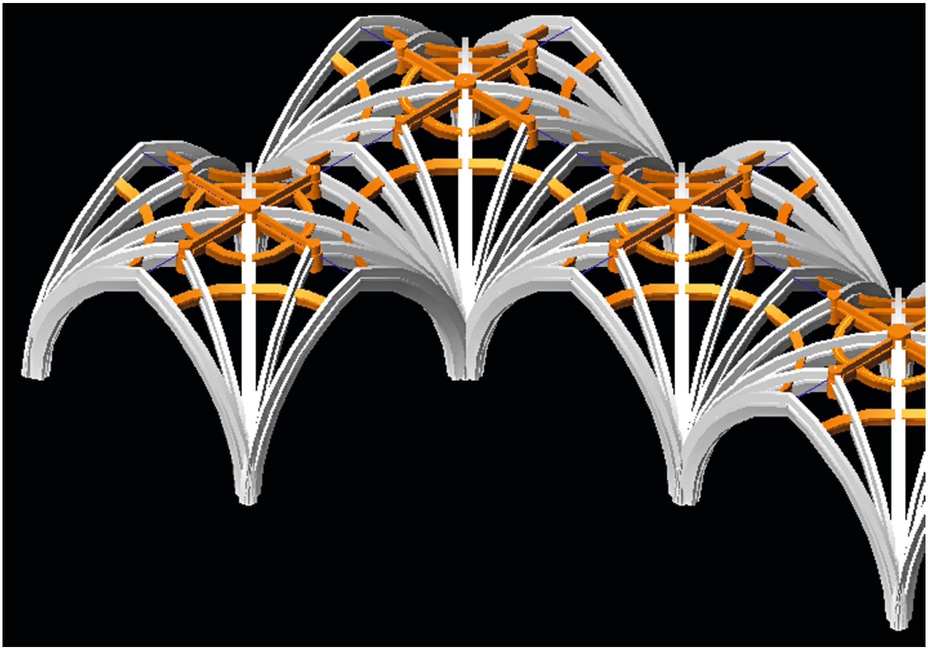

Fig. 27. Convent of San Esteban, Salamaca, cloister. Spectacular assembly of ribbed vault in the corner of the cloister

We previously noted that one of Juan de Álava's outstanding characteristics is his special way of laying out the subsidiary ribs so that they link the vaults of adjacent bays to generate an impression of continuity; this effect is enhanced when the section of the transverse arch is reduced to make it similar to the rest. Fig. 27 allows us to see the effect obtained with this type of subsidiary ribs and visualize the trumpet bell shapes of ribs created by this type of vaults from the springing to the ridge line lierne. This is particularly remarkable when the vault turns the corner in the cloister, where the springings of three of these vaults converge.

8 Flat Vaults

Flat vaults were very popular in Spain, more more than in the surrounding European countries, due to the emergence of a very particular typology of convent churches in Spain, in which the choir is situated on a high gallery over the first bays of the nave. Let us recall that, whereas in most European churches the choir was located in the presbytery, in Spanish cathedrals the choir occupied the central part of the main nave; nevertheless, in convents, the choir has a new location at the far end of the central nave, on an elevated storey through which one enters the church. This generally large, horizontal space is built on top of some vaults which needed to be surbased in order not to make this level excessively high and permit the visual communication between this platform and the high altar.

The huge expansion of religious orders in fifteenth- and sixteenth-century Spain increased the development of this type of church, and, consequently, the use of surbased vaults built with rib work. As we will now see, the layout of ribs in these vaults can be approached in two different ways: either through the use of very surbased basket-handle arches, or the use of segmental arches. In both cases, we come across the objective of reaching the highest possible standardization and simplification of the number of arches.

Convent of San Marcos, León. Vaults of the *Sotocoro.* Built between 1531 and 1538

In the first place let us see how to construct out a flat vault by using basket-handle arches, that is, three-centered arches or portions of these. One example of such vaults are those which support the impressive choir of San Marcos en León, built by Juan de Álava between 1531 and 1538 [Castro 2002] (fig. 28).

Fig. 28. Convent of San Marcos, León, flat vault under the elevated choir

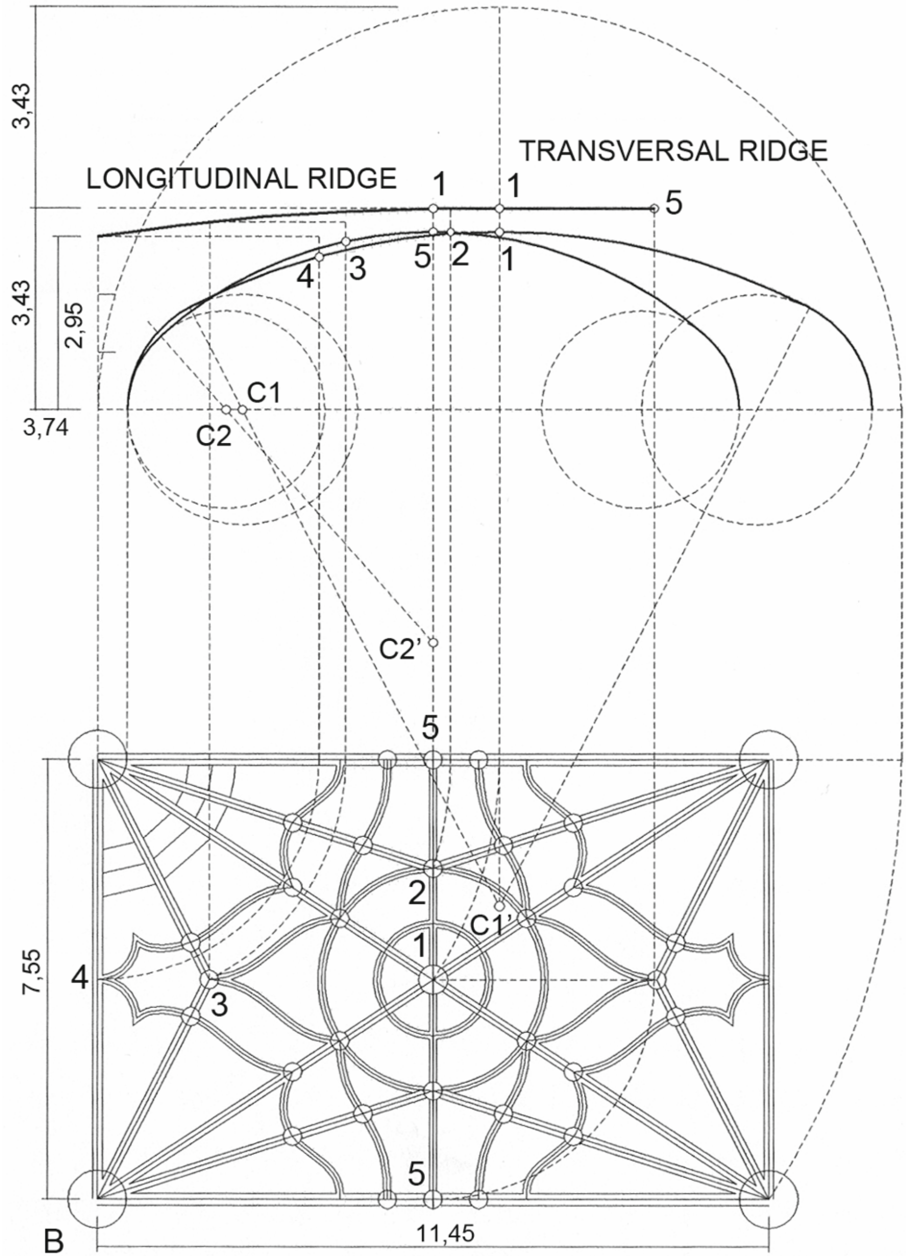

Fig. 29. Convent of San Marcos, León, vault of the *sotocoro*. The drawing shows how the curvatures of its ribs are obtained. All curvatures of the ribs, except the transversal arch, are identical to that of the three-centered diagonal arch. Once again, all ribs have been designed with the same curvature

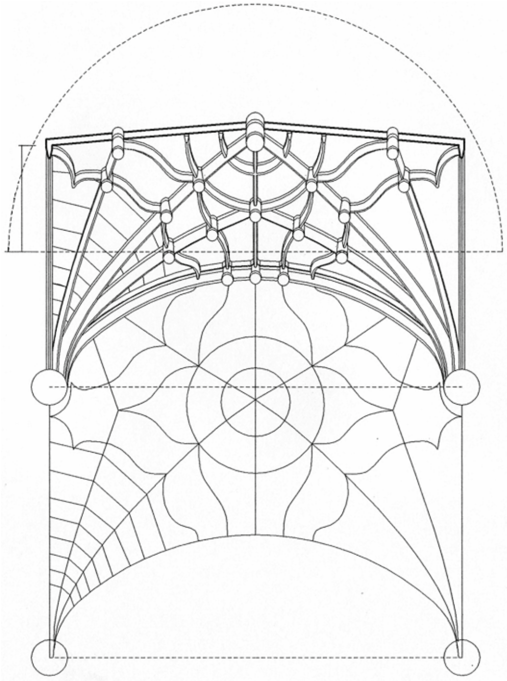

Fig. 30. Convent of San Marcos, León, vault of the *sotocoro*.
Prospective section showing the surbased profile of the vault obtained with the oval diagonal rib, exactly a quarter of the span of the vault

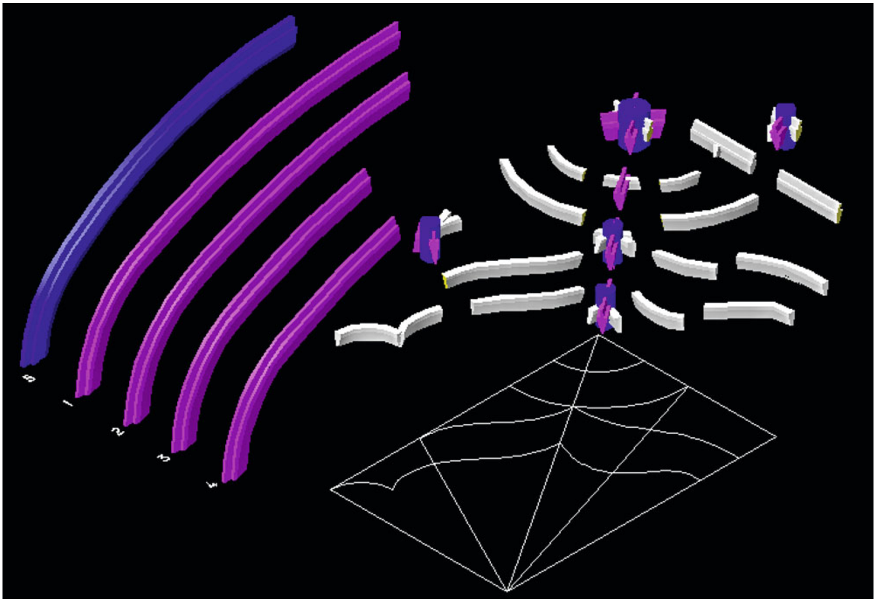

Fig. 31. Convent of San Marcos, León, vault of the *sotocoro*.
Standard parts required for the construction of this vault. All ribs, except the transversal arch, are obtained by cutting the diagonal rib

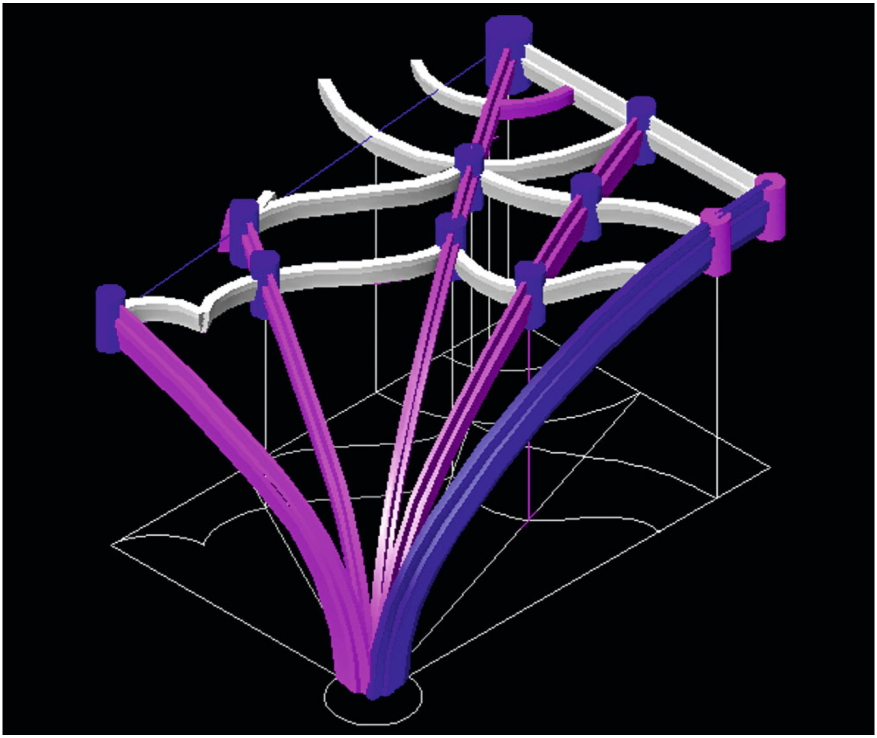

Fig. 32. Convent of San Marcos, León, vault of the *sotocoro*.
3D reconstruction of a quarter of the vault

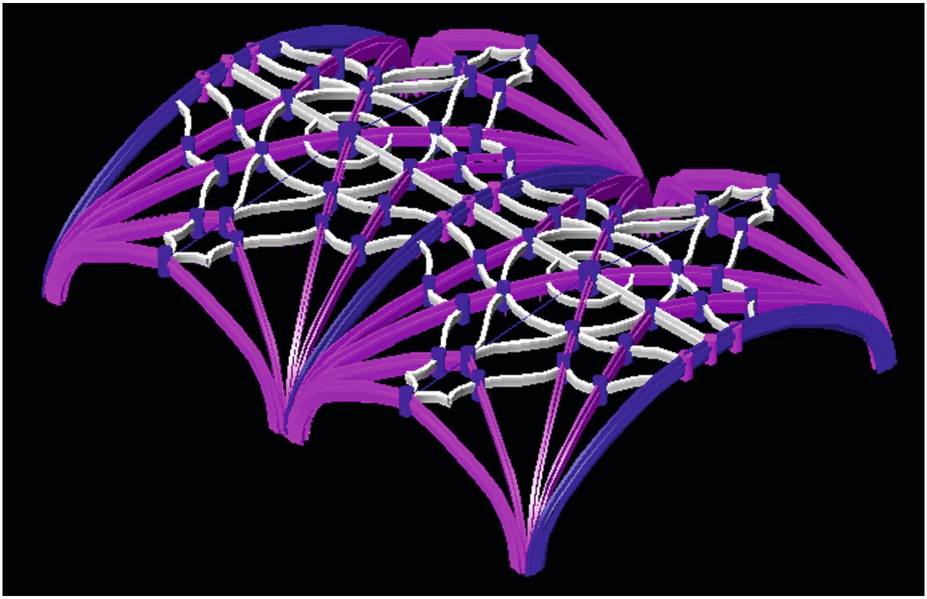

Fig. 33. Convent of San Marcos, León, vault of the*sotocoro*.
Volumetric view of two bays of the *sotocoro* showing the intricate grid of ribs

This is a rectangular plan vault which is due to a strict modulation; the sides of the rectangle are in the ratio of 3:2; we are thus dealing with a *sesquiáltera* proportion. Its sophisticated design responds to a regular grid of 6x4 modules, which makes it possible to situate the most important bosses, while the others can be located by alignments. This rib work serves two aesthetic purposes: on one hand, a central design is established, underlined by double circles; on the other, the subsidiary ribs create another set of ribs which, terminating abruptly in the transverse arch, will create a continuous web interlinking the vaults. We have already noted Juan de Álava's particular way of carrying out the rib work.

Beginning to draw the curvatures of the arches (fig. 29), let us drop the diagonal and draw a semicircular arch in elevation as a reference. We will, then, locate the central boss in its precise height, and we immediately verify that this boss is located at the midpoint of the radius, so the transverse arch will be half as high as the semicircular arch.

In this case, the springings of the transverse arches are located at the impost level, so the axis of the oval described by this arch maintains a 2:1 ratio. The oval of the diagonal has been drawn with the centres C1 and C1′, obtained by approximation. This basket-handle arch will be the one that generates the whole vault.

From the point 1, the central boss, can be drawn the two ridge lines of the vault. The smaller axis is a straight and horizontal line, whereas the rampant of the upper axis is a line, of a light curvature, descending with a slight slope. These two lines determine the height of the rest of the bosses.

When drawing the tiercerons 2, 3 and the formeret 4, we verify that their bosses are located precisely on the diagonal arch, which means that the three arches are portions of this arch. Therefore, the vault is designed in such a way that the layout of each one of them is obtained by cutting the diagonal arch with the span of the various arches. We thus find four equal arches; needless to say, this shows the obvious desire of the architect to rationalize the processes of drawing, cutting and centering of this sophisticated rib work.

It could be argued that, if the rampant in the longitudinal direction of the nave is horizontal, the tierceron 2 cannot be, strictly speaking, a portion of the diagonal arch; nevertheless, since the diagonal is such a surbased arch, its top remains practically horizontal for quite a long distance, which is sufficient to carry out the cut with which is obtained tierceron 2. In any case, it could be done rotating the arch's upper part onto the lower part, as we noticed when talking about lierne vaults.

The transverse arch, on the contrary, is different; its boss is at the same height as the central boss and its span is that of the vault's larger side, so this arch requires a specific oval. The basket-handle arch drawn with the centres C2 and C2′ seems to adjust to these requirements.

Thus, such an apparently complex vault can be built with only two different arches: the diagonal and the transverse arch. Fig. 30 shows a perspective section in which one can observe the gradient of the longitudinal *rampant*, the line derived from progressively cutting the diagonal arch with the span of tierceron 3 and the formeret 4. In the figure, we can also see the panels laid in horizontal courses, "in the English way".

As shown in Fig. 31, we can envision the set of pieces required to carry out the vault: first, the collection of arches which, except the transverse arch, are clearly portions of the diagonal arch, that is to say equally traced. Then, one can see the six different bosses required by the vault and the set of subsidiary ribs which are fragments of flat ribs. As can be seen in fig. 32, with this repertory of pieces a quarter of the vault could be built.

Fig. 33 is a model of this vault in which we can well appreciate the resulting volumetry and its complex rib work. The image shows the spatial grid formed by the ribs of the three vaults making up the *sotocoro*, all of them built with arches generated from the diagonal's oval. It also allows us to verify the great skill of Gothic architects in the sixteenth century in solving these imposing three-dimensional webs.

San Esteban Convent, Salamanca. *Sotocoro* vaults. Built between 1524 and 1610

The vault of the *sotocoro* in the convent of San Esteban de Salamanca (fig. 34) was built by Juan de Álava. It is a very surbased flat vault obtained by using oval arches of a very low rise. It has a rectangular plan measuring approximately 15 x 7.5 m; fig. 35 shows its complex plan and the ridge of its two sections. For the longitudinal rampant (in the direction of the nave axis), Juan de Álava decided on a straight, horizontal line, which means both the central boss and that of the transverse arch must be at the same height, but in the transversal direction he fixed a slightly curved rampant with a drop towards the boss of the formeret arch. These two lines are essential, since they precisely define the shape of the vault as well as determine the height of the secondary bosses (fig. 36).

Fig. 34. Convent of San Esteban, Salamanca.
Flat vault of the sotocoro built by Juan de Álava

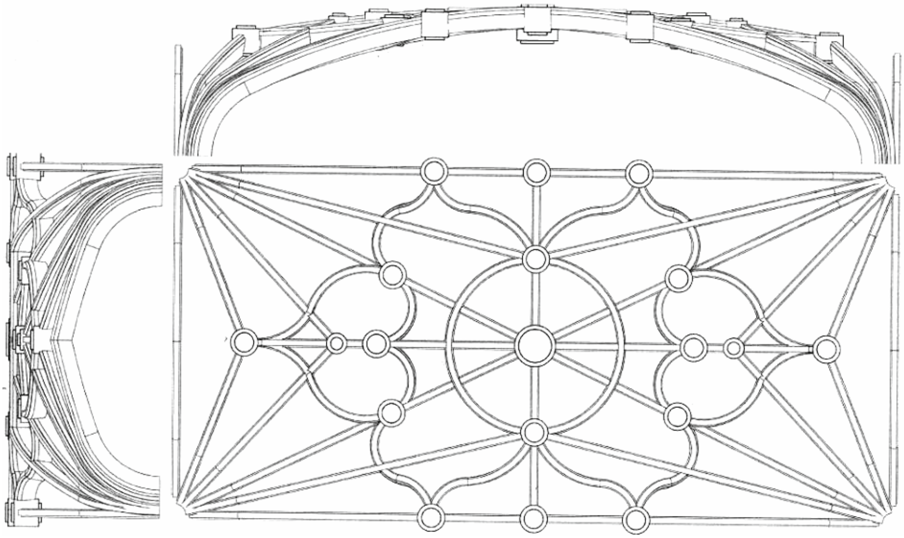

Fig. 35. Convent of San Esteban, Salamanca, flat vault of the *sotocoro*. Plan and sections of the vault showing its horizontal longitudinal section and its cross section curved and inclined over the formeret arches

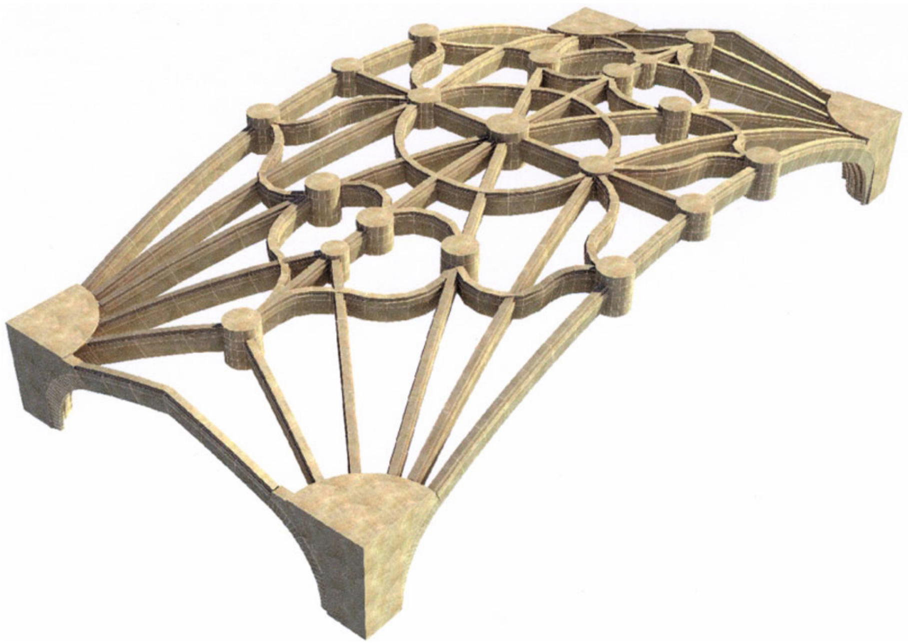

Fig. 36. Convent of San Esteban, Salamanca. 3D reconstruction of the vault of the high choir of the (drawing by Jorge Cerdá)

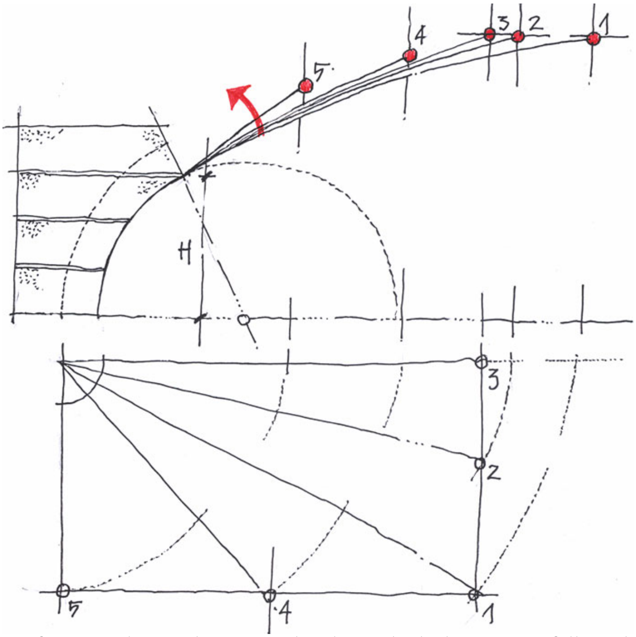

Fig. 37. Convent of San Esteban, Salamanca. Sketch in which the strategy followed by the architect to standardize the curvatures of the arches is shown. Each vault is laid out using the upper part of the diagonal oval arch

In order to fix the curvature of the tiercerons, Juan de Álava seems to have resorted to the strategy summed up in the sketch shown in fig. 37. In the first place, the lower part of the oval, which is not a real arch, coincides with the *tas-de-charge* of the vault and is built with horizontal beds. These courses rise to the height of the tangency point between the upper and the lower curvature of the diagonal oval, which had been drawn with the height of the central boss (point 1) deemed appropriate by the architect. In the second place, Juan de Álava decided that all the tiercerons should be oval arches whose curvatures are identical to that of the diagonal arch. Then, two conditions which will remarkably simplify the vault's construction are set. First, the lower part of the arch of the tiercerons is made to coincide with that of the diagonal arch; second, the height of the *tas-de-charge*, which fixed the diagonal arch H, is made identical in all the arches. Subject to these conditions, the various tiercerons are easily drawn, rolling the upper part of the diagonal arch from point H until it coincides, at the other end, with the height of each of the bosses. All things considered, the vault has been built with just one arch: the upper part of the oval of the diagonal rib.

At the Escuela de Arquitectura de Madrid, we had the opportunity to verify the hypothesis proposed in the previous paragraphs by building a scale model of this vault; the experience allows us to assert the accuracy of the precedent considerations. We proceeded first to lay it out at full-scale in plan and elevation (fig. 38). Following the patterns that we know of medieval stonecutting workshops, we extracted the data necessary to carve of each of the pieces that make up the vault (fig. 39). Once the carving process finished, we proceeded to the assembly of the vault, placing the pieces previously carved on a wooden centering (fig. 39). It is obvious that the centerings, all of a single curvature, are extremely simplified by the information provided. The final result is a scale model of the vault confirming the proposed hypothesis (fig. 40).

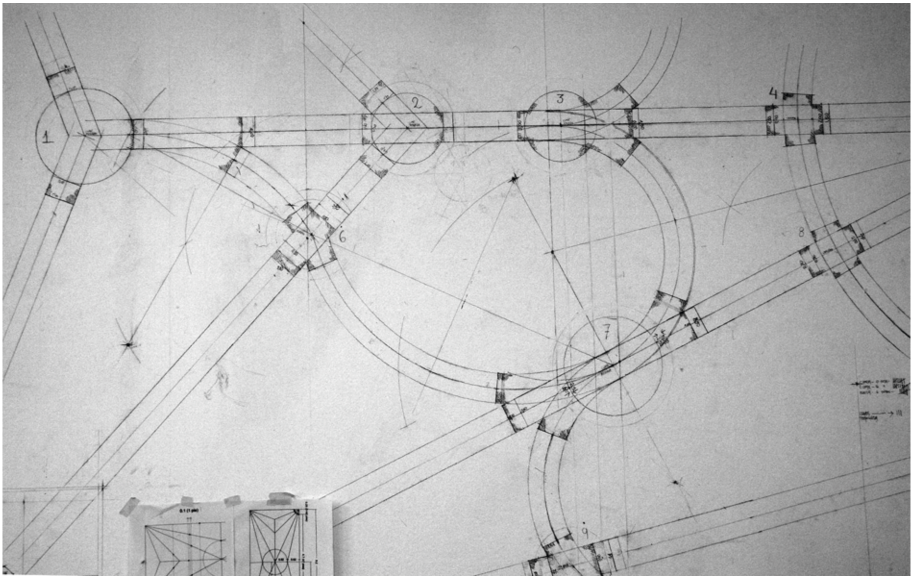

Fig. 38. Full-scale tracings drawn on the wall destined for the construction of a reduced-scale model of the vault

Fig. 39. Set-up; the different pieces composing the vault are placed on the centerings. The vault was built with only one radius of arch

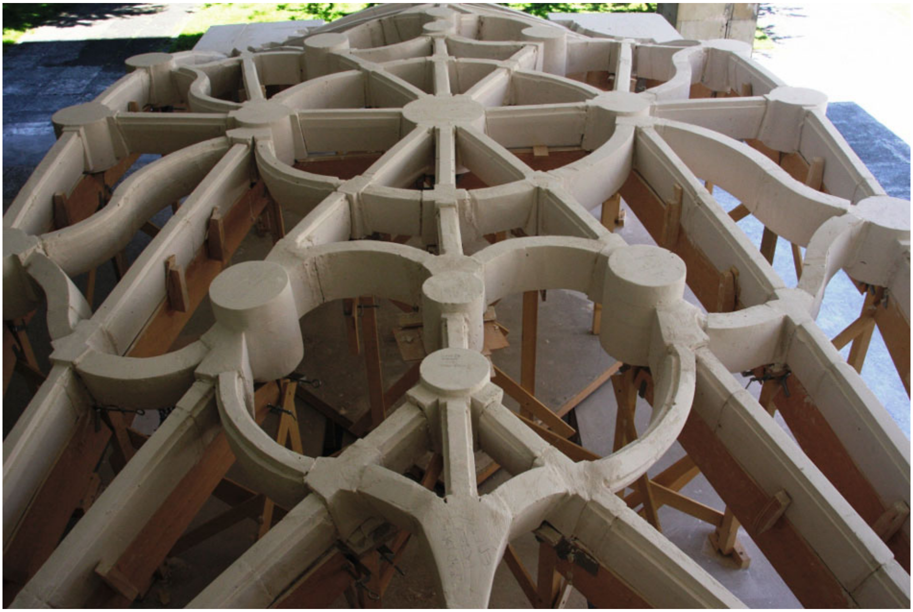

Fig. 40. Reproduction of the sophisticated vault by Juan de Álava. This model was built at the School of Architecture of Madrid at a scale of 1:3, in the 2010 academic year

9 Conclusion

From its origins, the Gothic vault was conceived as an arrangement of cells whose folds are curved lines materialized in stone ribs. Thus, the Gothic vault of complex rib work can be described as a set of interconnected curved lines which forms a spatial grid; the vault is then formed when a thin shell of panels is spread over that grid. In short, Gothic stonemasonry relied on the definition of lines, and, to that purpose, created a geometric instrument that made it possible to carry out that task. It was with that goal in mind, and at that particular moment, when, during the long winters of inactivity and at the drawing boards, were generated the rudiments of the projection system nowadays known as dihedral, that is, a geometric method for establishing exactly the concordance between the plan and the elevation.

One result of that idea is the notion that there is no form of Gothic vault established a priori, but rather its form results from the spatial grid we just referred to. The grid's shape is a result of placing a series of points – the bosses – in space; these points are then connected by arches which are sometimes drawn from the vault's springing and sometimes from other bosses.

Gothic art is subject to a degree of increasing complexity; the subdivision of any of its elements has an influence on its vaults and the system is capable of drawing constructive lessons from that aesthetic drive. The increasing fragmentation of the panel cells results in their being easier to build; it is interesting to note how the multiplication of ribs is carried out according to very intelligent criteria for homogeneity and standardization.

In contrast, the Renaissance vault relied on perfectly defined form and was adjusted to pure prismatic shapes, always perfect from the geometric point of view, whether they are surfaces of revolution, toruses, ruled or prismatic surfaces. However, the breakdown of these surfaces into voussoirs results in blocks of great volumetric complexity requiring a more developed geometry, one which, though based on the same foundations as the Gothic, goes beyond that in order to understand and define very complex volumes, such as the voussoir work created in Renaissance vaults. From that time, on one can talk of stereotomy, that is, the cut (*tomos*) of the space (*estéreo*).

We can, thus, sum up by saying that the stonemasonry of the Gothic vault in its totality is based upon geometry of the line, whereas Renaissance stereotomy relies on the comprehensive knowledge of the surface and the highly sophisticated surfaces of the voussoirs necessary for its vaults. But it is obvious that this leap in the art of construction was paralleled and accompanied by an extension of the horizons of geometry. In Spain, it could be carried out thanks to the centuries-old tradition of building in stone that began in the most remote medieval times, and to the presence of architects and stonemasons as outstanding as Juan de Álava, whose professional work exceeded the established limits and provided the art of building with new instruments.

Acknowledgments

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References

ARAUJO, Sebastián and Jaime NADAL. 1997. *Las catedrales de Plasencia, Historia de una restauración*. Cáceres : Colegio Oficial de Arquitectos de Extremadura.

BECHMANN, Roland. 1981. *Les racins des cathédrales.* Paris: Payot.

———. 1991. *Villard de Honnecourt, la pensée technique au XIII siècle et sa communication*. Paris: Picard.

CASTRO SANTAMARÍA, Ana. 2002. *Juan de Álava, arquitecto del Renacimiento*. Salamanca: CajaDuero.

CALVO LÓPEZ, José. 2009. La literatura de la cantería: una visión sintética. Pp. 101-156 in *El arte de la piedra.* Madrid: CEU Ediciones.

CHOISY, Auguste. 1899. *Histoire de l'architecture.* 2 vols. Paris: Gauthier-Villar.

CHUECA GOITIA, Fernando. 2001. *Historia de la arquitectura española*. 2 vols. Ávila-Madrid: Fundación Cultural Sta. Teresa: C.O.A.M Ávila.

FRANKL, Paul. 1962. *Gothic Architecture*. Harmondsworth: Pelican Books.

GARCÍA, Simón. 1991. *Compendio de arquitectura y simetría de los templos* (1681). Introductions by de Antonio Bonet Correa and Carlos Chanfón Olmos. Valladolid: Colegio Oficial de Arquitectos de Valladolid.

GÓMEZ MARTÍNEZ, Javier. 1998. *El gótico español en la Edad Moderna: bóvedas de crucería*. Valladolid: Universidad.

MERINO DE CÁCERES, José Miguel. 1996. *El claustro de la catedral de Segovia*. Segovia: Diputación Provincial.

MÜLLER, Werner. 1990. *Grundlagen gotischer bautechnik: ars sine scientia nihil est*. Munich: Deutscher Kunstverlag.

NAVASCUÉS PALACIO, Pedro. 1984. *Monasterios de España*. Madrid: Espasa Calpe.

PALACIOS GONZALO, José Carlos. 2003. *Trazas y Cortes de Cantería en el Renacimiento Español*. Madrid: Munilla-Lería.

———. 2009. *La cantería medieval, la construcción de la bóveda gótica española*. Madrid: Munilla Lería.

———. 2010. "Descimbrado de la bóveda gótica de san Esteban". YouTube. Madrid: UPM.

PALACIOS, José Carlos and Rafael MARTÍN. 2009. La construcción de una bóveda de crucería en la Escuela Técnica Superior de Arquitectura de Madrid. *Informes de la Construcción* 61, 515: 49-58.

RABASA DIAZ, Enrique. 2000. *Forma y Construcción en piedra, de la cantería medieval a la estereotomía del siglo XIX.* Madrid: Akal.

———. 2005. Construcción de una bóveda de crucería en el centro de oficios de León. Pp. 909-917 in *Actas del cuarto Congreso de Historia de la Construcción,* Cádiz, vol II. Madrid: Instituto Juan de Herrera.

VIOLLET-LE-DUC, Emmanuel. 1854-1868. *Dictionnaire raisonné de l'architecture française du XI au XVI siècle*, 10 vols. Paris: B. Banche Editeur.

———. 1878. *Histoire d'une hôtel de ville et d'une cathédrale*. Paris: J. Hetzel. Rpt. Bruxelles: Pierre Mardaga, 1978.

WENDLAND, David. 2009. *Lassaulx und der Gewölbebau mit selbsttragenden Mauerschichten. Neumittelalterliche Architektur um 1825-1848*. Petersberg: Imhof.

Willis, Robert. 1842. *On the constructions of the Vaults of the Middle Ages*. London. Rpt. Transactions of the RIBA, Longman, Brown, Green and Longmans, 1910.

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